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THIS JOURNAL IS DEDICATED TO THE FOLLOWING AIMS: (1) Through published standard papers on the culture aspects, humanism and history of mathematics to deepen and to widen public interest in its values. (2) To supply an additional medium for the publication of expository mathematical articles. (3) To promote more scientific methods of teaching mathematics. (4) To publish and to distribute to groups most interested high-class papers of research quality representing all mathematical fields.

Every paper on technical mathematics offered for publication should be submitted (with enough enclosed postage to cover two two-way transmissions) to the Chairman of the appropriate Committee, or to a Committee member whom the Chairman may designate to examine it, after being requested to do so by the writer. If approved for publication, the Committee will forward it to the Editor and Manager at Baton Rouge, who will notify the writer of its acceptance for publication. If the paper is not approved the Committee will so notify the Editor and Manager, who will inform the writer accordingly.

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More About the Applicative Phase of Mathematics

A mathematical correspondent—A. Judson Moore—writing from Fort Worth, Texas about our April editorial remarks, "It was too short and left too much unsaid." The reflections of a reader may sometimes make it unnecessary to state all the implications of a proposition. But if hypotheses are invalid, it may well be that implications are invalid. Then, too, come questions: What hypotheses are valid for all minds alike? If none such exist, who shall be judge of what is valid and what not valid in one's assumptions about anything?

(a) A fundamental assumption implicit in the April essay was that Truth (capital T), defined or undefined, has the property of universality, such property not needing to be specifically defined.

(b) A second assumption, also implicit, was the inseparability of Truth and reality. By our first assumption this reality must be universal and hence inclusive of all humanity, warring or at peace, happy or unhappy, rich or starving. From this last it follows that Truth (with capital T) is not being envisioned by the solitary mathematician when he merely sees a consistency between data and deductions (truth with a small t) in a limited area of reality or fancy.

Says Cassius J. Keyser in his *The Human Worth of Rigorous Thinking*, "The crtical mathematician has abandoned the search for truth. He no longer flatters himself that his propositions are or can be known to him or to any other human being to be true; and he contents himself with aiming at the correct, or the consistent."

S. T. Sanders.

Sturm's Theorem for Multiple Roots

By Joseph Miller Thomas
Duke University

In stating Sturm's theorem for a polynomial $f(x)$ it is usual to suppose that f and its derivative f' are relatively prime. Some books, in addition, state a theorem applicable to polynomials with multiple roots.* The ordinary form of this theorem gives the number of distinct roots but does not determine the multiplicity of the roots. The purpose of this note is to indicate two theorems each of which determines the multiplicity. The first of these may be regarded as the more nearly complete because it is stated for a point set of the form $a<x\leqq b$. The second is not applicable to an interval if one of the end points is a root, but it is in general easier to apply because it involves fewer divisions.

Denote by $f(x)=f_0(x)$ a polynomial with real coefficients and by $f_1(x)$ its derivative. Apply Euclid's division algorithm to f_0, f_1, changing the sign of each remainder, and so form the sequence

$$f_0, f_1, f_2, \cdots, f_l, \tag{1}$$

the last member of which is a highest common factor of f_0, f_1.

We first outline a proof of Sturm's theorem for a polynomial without multiple roots. The relation

$$f_{i-1}=f_iq_i-f_{i+1} \qquad (i>0)$$

and the fact that f_0, f_1 are relatively prime are used to prove

(i) Sequence (1) can suffer a change in the number of variations of sign only at a root of f_0.

This result can be applied to the subsequence obtained by omitting the first term of (1) because the subsequence is generated by the division algorithm applied to f_1, f_2 which are relatively prime. Hence the subsequence can have a change in the number of variations only at a root of f_1. Since no root of f_0 is a root of f_1 we have

(ii) The subsequence obtained by omitting the first term f_0 from (1) does not suffer a change in the number of variations of sign at a root of f_0.

*If $f(a)=0$, then a is called a root of the polynomial $f(x)$ as well as of the equation $f(x)=0$.

We now use for the first time the fact that f_1 is the derivative of f_0. As x increases through a root, f_0^2 decreases to zero and then increases, that is, the derivative of f_0^2 is negative just before a root and positive just after. Hence we see that

(iii) The product f_0f_1 is negative just before a root of f_0 and positive just after.*

Because of (iii), just before a root of f_0 the first two terms of (1) show a variation in sign, which is lost at the root because f_0 becomes zero at the root and has the same sign as f_1 just after. This fact coupled with (i) and (ii) shows that the loss in variations on the set $a<x\leqq b$ is the number of real roots on that set.

Now suppose that f_l is not necessarily constant. Let the members of (1) be divided by f_l to give

$$g_0,\ g_1,\ g_2,\ \cdots,\ 1. \tag{2}$$

This sequence will be called the *first Sturm sequence for* f_0. It can be generated from g_0, g_1 in the same way as (1) is generated from f_0, f_1. Note, however, that g_1 is not in general the derivative of g_0.

Since (2) is generated by the division algorithm from relatively prime g_0, g_1, statements (i) and (ii) are true if applied to (2) and to g_0. Moreover, since

$$f_0f_1=f_l^2g_0g_1,$$

we see that g_0g_1 is negative just before a root of f_0 and positive just after. The roots of g_0 are the distinct roots of f_0. Hence the loss in variations of (2) on $a<x\leqq b$ is the number of distinct roots of f_0 on that set.

If f_l is not constant, the first Sturm sequence for f_l will be called the *second Sturm sequence* for f_0. It is formed, of course, by applying the division algorithm to f_l, f_l'.

In general, if φ_k is the factor removed by division in obtaining the kth Sturm sequence, the first Sturm sequence for φ_k is the $(k+1)$th Sturm sequence for f_0. The process of forming successive sequences is halted when the highest common factor like f_l becomes constant. Let $S_k(x)$ denote the number of variations of sign in the kth Sturm sequence for the value x and write

$$S(x)=S_1(x)+\cdots+S_p(x),$$

where the last Sturm sequence is the pth.

*This useful theorem was stated by A. Hurwitz in discussing a generalization of Budan's theorem. See his paper *Ueber den Satz von Budan-Fourier*, Mathematische Annalen, Vol. 71 (1912), pp. 584-591; p. 585.

The distinct roots of φ_k are the roots of f_0 whose multiplicity is at least $k+1$. The loss in variations of the kth sequence accordingly is the number of roots of f_0 whose multiplicity is at least k. The number of roots whose multiplicity is exactly k can be found by subtraction. If we interpret S_q as zero for $q>p$, the number of roots of multiplicity k on $a<x\leqq b$ is

$$S_k(a)-S_{k+1}(a)-[S_k(b)-S_{k+1}(b)].$$

Moreover, the following generalized form of Sturm's theorem is seen to be true.

Theorem 1. *The number of roots on $a<x\leqq b$ is $S(a)-S(b)$, a root of multiplicity k being counted as k roots.*

If f_l is positive, a member of (1) is $+$, 0, or $-$ according as the corresponding member of (2) is $+$, 0, or $-$. If f_l is negative, a member of (1) is $+$, 0, or $-$ according as the corresponding member of (2) is $-$, 0, or $+$. In either case, the number of variations in (1) is the same as in (2). If, as is easy to do in actually locating roots, we choose intervals neither end of which is a root, we may employ the sequences like (1) and so avoid the divisions necessary in passing to the actual Sturm sequences like (2). For this purpose, it seems desirable to place all the f's in a single sequence, whose generation can be described as follows:

(i) If the remainder in the division $f_{j-2}\div f_{j-1}$ is not zero, that remainder with its sign changed is taken as f_j.

(ii) If the above mentioned remainder is zero and if f'_{j-1} is not zero, then f'_{j-1} is taken as f_j.

(iii) If f_{j-1} is a non-zero constant, the sequence is stopped at f_{j-1} and there is no f_j.

The sequence so obtained will be called the *extended Sturm sequence*. Evidently we have

Theorem 2. *If $f(a)f(b)\neq 0$ and if a root of multiplicity k is counted as k roots, the number of roots on the interval a, b is the same as the loss in variations of the extended Sturm sequence.*

The following example concerns a polynomial with three Sturm sequences. In applying the division process, positive numerical factors have been introduced where convenient. The three sequences of f's are written at the left. Corresponding Sturm sequences are

written opposite them at the right. Taken together, all the f's constitute the extended Sturm sequence.

$$\begin{array}{ll} f_0 = x^6 - 4x^5 + 5x^4 - 2x^3 & g_0 = x^3 - 3x^2 + 2x \\ f_1 = 3x^5 - 10x^4 + 10x^3 - 3x^2 = \frac{1}{2}f' & g_1 = 3x^2 - 7x + 3 \\ f_2 = 5x^4 - 11x^3 + 6x^2 & g_2 = 5x - 6 \\ f_3 = x^3 - x^2 & g_3 = 1 \\ & \\ f_3 = x^3 - x^2 & h_0 = x^2 - x \\ f_4 = 3x^2 - 2x & h_1 = 3x - 2 \\ f_5 = x & h_2 = 1 \\ & \\ f_5 = x & k_0 = x \\ f_6 = 1 & k_1 = 1. \end{array}$$

The signs of the first f sequence for $-\infty, \infty$ are respectively

$$+ - + -, \qquad + + + +,$$

so that f_0 has three real, distinct roots. The signs of the three Sturm sequences for the same values are

$$\begin{array}{ll} - + - + & + + + + \\ + - + & + + + \\ - + & + + \end{array}$$

Accordingly, there are $3+2+1$ real roots, of which one is simple, one is double and one is triple.

The extended sequence consists of $f_0, \cdots, f_6$. Its loss of variations in sign in passing from $-\infty$ to $+\infty$ is six.

At Harvard University Mr. Lynn Harold Loomis of the Society of Fellows has been appointed Faculty Instructor in Mathematics.
—Reported by L. J. Adams.

The theory of infinite series from the standpoint of a sophomore is frequently productive of startling results. From a test paper comes: "An alternating series is a series in which the signs $1-1+1-1\cdots$, are alternatingly changed as shown by the illustration; if this series ends when infinity is even, then the series is convergent, the sum being zero; if it ends when infinity is odd, then the sum is one."

A conic section the Greeks didn't dream of is found in this excerpt from an analytics test: "An elipse is the locus of all points whose distance from a fixed point is equal to its distance from a fixed constant less than one."

Formulas Suitable for Machine Computation for the Mean, Standard Deviation, and Coefficient of Correlation for a Composite Group in Terms of Similar Indices of Its Subgroups

C. N. Mills
Illinois State Normal University, Normal, Illinois

In certain types of statistical problems it is often desirable to determine the statistical elements of a composite group in terms of the statistical elements of its subgroups. This discussion is concerned with the mean, standard deviation, and coefficient of correlation.

Weighted Mean. It is often desirable to compute sample averages from the averages of certain subgroups of the sample. The well known formula for the mean is

$$NM = \sum_{i=1}^{i=n} N_i M_i, \tag{1}$$

where N_i and M_i are the number and mean respectively of the ith subgroup and n is the number of the subgroups.

Weighted Standard Deviation. The following forumla is a slight variation of a formula given by Baten* and others, arranged here for convenience in machine computation,

$$N_T \sigma_T^2 = \sum_{i=1}^{i=n} N_i \sigma_i^2 + \sum_{i=1}^{i=n} N_i M_i^2 - N_T M_T. \tag{2}$$

Weighted Coefficient of Correlation. The averaging of coefficients of correlation of subgroups, and of samples, to determine the coefficient of correlation of the total group, or population, has been questioned

*Baten, D. W., Mathematical Statistics, p. 75.
Yule, G. U., Introduction to the Theory of Statistics, (Eighth Edition) p. 142, formula 7.
Holzinger, Karl, Statistical Methods in Education, page 115, formula 21.
Garrett, H. E., Statistics in Psychology and Education, (Revised Edition 1938), p. 192, formula 25.
Kirschmann, A., Grundzuge der Psychologischen Maszmethoden, in Abderhaldens Handbuch der Biologischen Arbeitsmethoden, Abt. VI, Teil A, p. 409, formula 10.

on the basis of statistical accuracy. In educational literature we find that it is a common practice to find averages. Unless the subgroups satisfy tests for conformity to the total group, the averaging of the coefficients is not statistically justified. The coefficients do not vary along a linear scale, which means that an increase from .50 to .60 does not mean the same increase in degree of relationship as an increase from .80 to .90. Also some of the coefficients may be positive and negative. Thus the average of +.70 and −.70 is .00, indicating no real relationship. When comparable groups are considered, and the coefficients do not differ greatly in value, it seems justifiable to average the coefficients. However, the best plan is not to use the average method.

The following formula makes use of the raw scores, and is suitable for machine computation,

$$rN_T\sigma_X\sigma_Y = \sum_{i=1}^{i=n} r_iN_i\sigma_{x_i}\sigma_{y_i} + \sum_{i=1}^{i=n} N_iM_{x_i}M_{y_i} - N_TM_XM_Y. \tag{3}$$

Derivation of the Formula. Let a_1, a_2, a_3, etc. and b_1, b_2, b_3, etc. be the paired items in the first subgroup (X_1, Y_1); c_1, c_2, c_3, etc. and d_1, d_2, d_3, etc. be the paired items in the second subgroup (X_2, Y_2), and so on.

For the first subgroup

$$r_1 = \frac{\dfrac{\Sigma ab}{N_1} - M_aM_b}{\sigma_a\sigma_b}. \tag{4}$$

For the second subgroup

$$r_2 = \frac{\dfrac{\Sigma cd}{N_2} - M_cM_d}{\sigma_c\sigma_d}, \tag{5}$$

and so on. From (4) and (5) we get

$$\begin{cases} \Sigma ab = N_1r_1\sigma_a\sigma_b + N_1M_aM_b, \\ \Sigma cd = N_2r_2\sigma_c\sigma_d + N_2M_cM_d, \\ \text{and so on.} \end{cases} \tag{6}$$

For the total group of paired items we know that

$$r = \frac{\dfrac{\Sigma ab + \Sigma cd + \cdots}{N_T} - M_XM_Y}{\sigma_X\sigma_Y}.$$

Hence

$$\Sigma ab + \Sigma cd + \cdots = rN_T\sigma_X\sigma_Y + N_TM_XM_Y. \tag{7}$$

The sum of the right members of (6) is equal to the right member of (7). Then

$$rN_T\sigma_X\sigma_Y = r_1N_1\sigma_{x_1}\sigma_{y_1} + r_2N_2\sigma_{x_2}\sigma_{y_2} + \cdots$$
$$+ N_1M_{x_1}M_{y_1} + N_2M_{x_2}M_{y_2} + \cdots - N_TM_XM_Y,$$

or

$$rN_T\sigma_X\sigma_Y = \sum_{i=1}^{i=n} r_iN_i\sigma_{x_i}\sigma_{y_i} + \sum_{i=1}^{i=n} N_iM_{x_i}M_{y_i} - N_TM_XM_Y.$$

Example

The data for this example have been determined by the scores on the Teachers College Psychological Examination (1939 Form) and the Vinson English Test. The tests were administered to 600 freshmen at Illinois State Normal University in September, 1939. The 600 papers were arranged into subgroups of 100 papers, 200 papers and 300 papers. The mean, standard deviation, and the coefficient of correlation have been determined by the use of the recommended formulas. All computations have been approximated to the nearest third decimal figure. The following tables give the elements entering into the formulas, and the corresponding elements for the total group.

Table 1

Psychological Examination

	N	*Mean*	*SD*
	100	65.53	25.91
Subgroups	200	62.43	21.45
	300	64.01	23.79
Total Group	600	63.74	23.44

Table 2

English Test

	N	*Mean*	*SD*
	100	84.99	26.72
Subgroups	200	86.97	24.73
	300	87.25	24.08
Total Group	600	86.78	24.77

Table 3

Coefficients of Correlation

English Test and Psychological Examination

	N	r
	100	.646
Subgroups	200	.642
	300	.663
Total Group	600	.650

As a check upon all computations, data for the subgroups and for the total group, the Hull-Ayres method using the raw scores yielded results in agreement with the results as determined by the use of the recommended formulas.

The reader might wonder just why the total group of 600 was broken into the subgroups of 100, 200, and 300 respectively. In the study where the problem was met, it happened to fall into the respective categories. The total group might have been broken up into any number of subgroups.

Computational Savings in Routine Comparisons of the Means of Correlated Observations

By G. A. Baker
University of California

The significance of the difference between the means of two correlated series of observations can be tested by computing

$$v = \frac{(\sum d_i)^2}{n(\sum d_i^2)}, \tag{1}$$

where d_i = the difference between the corresponding observations and n = the number of pairs of observations, instead of the more cumbersome

$$t = \frac{(1/n)\sum d_i}{\sqrt{(1/n)\sum (d_i - (\sum d_i/n))^2}}. \tag{2}$$

The saving in computation in routine analyses is very considerable.

Let

$$y = (\sum d_i/n)^2, \quad x = 1/n(\sum (d_i - (\sum d_i/n))^2).. \tag{3}$$

Then (1) can be written as

$$v = \frac{y}{x+y}, \quad 0 \leq v \leq 1. \tag{4}$$

The joint distribution of x and y is known to be

$$\frac{((n/2)\sigma^2)^{(n-1)/2}}{2(2\pi\sigma^2/n)^{1/2}\Gamma(n-1/2)} \quad \frac{x^{(n-3)/2}}{\sqrt{y}} \, e^{-(nx/2\sigma^2)} e^{-(n/2\sigma^2)y} dx dy \tag{5}$$

for samples of n drawn at random from a population represented by

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} \, e^{-(z^2/2\sigma^2)}, \quad -\infty \leq z \leq \infty. \tag{6}$$

Make the transformation

$$x = \frac{1-v}{v} \, u, \quad y = u. \tag{7}$$

The Jacobian of the transformation (7) is u/v^2, Hence, v is distributed as proportional to

$$(4) \qquad 1/v^2 \int_0^\infty \left(\frac{1-v}{v} u \right)^{(n-3)/2} e^{-(nu/2\sigma^2)} \, e^{-\frac{n(1-v/v)u}{2\sigma^2}} \sqrt{u} \, du.$$

The probability distribution of v is

$$(9) \qquad \frac{\Gamma(n/2)}{\sqrt{\pi}\,\Gamma(n-1)/2} \, v^{-\frac{1}{2}}(1-v)^{(n-3)/2}.$$

Thus v given by (1) can be calculated and the hypothesis of a normal population of differences with zero mean can be tested by entering a table of the incomplete Beta functions instead of calculating t given by (2) and entering a table of the t-distribution.

Example: Consider the differences 1.8, 1.2, 0.8, 1.2, 1.4, 0.2. The value of t for these data is 4.92 giving a probability of 0.0046. For these data $v = 1.21/1.46 = 0.8288$. The probability of a value of v as large or larger than 0.8288 is

$$\frac{\Gamma(3)}{\sqrt{\pi}\,\Gamma(5/2)} \int_{0.8288}^{1} v^{1/2}(1-v)^{3/2} dv = 0.0046.$$

The Mil as an Angular Unit and Its Importance to the Army*

By RICHARD S. BURINGTON
Case School of Applied Science

In the Army of the United States, two systems for measuring angles are in wide use; the *mil* system and the familiar *sexagesimal* system. In the mil system, the fundamental unit is the *mil*, where by definition 1600 mils equal one right angle. In contrast, in the sexagesimal system, the fundamental unit is the degree, where 90 degrees equal one right angle. Radian measure is also used to some extent by the Army. Most American mobile artillery units as well as many heavy railway mounts have the scales on their sights, azimuth circles, and quadrants graduated in mils, and some of these units have their scales graduated in both mils and degrees. The mil is also employed to a large extent by the Infantry.

The *mil* gains its name from the fact that one mil is approximately the angle subtended by one yard at a distance of 1000 yards. This simple approximate relation makes the mil well adapted to certain types of practical calculations. It is principally for this reason that the mil system is used extensively in several branches of the army.

*Prepared at the request of the Sub-Committee on Education for Service of the War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America.

(*Continued on page 402*)

Hyperbolic Functions in Concentric Circles

By Marcus A. Roby, Sr.
Portland, Oregon

Consider the concentric circles

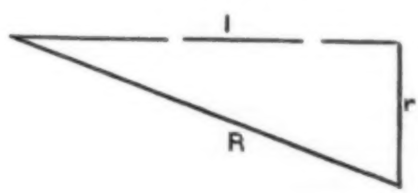
(1) $$x^2+y^2=r^2$$

(2) $$x^2+y^2=R^2$$

with $$R^2=1+r^2$$

$$A(r\cos\Theta,\ r\sin\Theta)$$

$$B(-r\cos\Theta,\ r\sin\Theta)$$



$$C(-\sqrt{R^2-r^2\sin^2\Theta},\ r\sin\Theta)$$

Let $$DA=\sinh u$$

$$CD=\cosh u.$$

This is possible since

$$CD^2=1+AD^2$$

$$R^2-r^2\sin^2\Theta=1+r^2\cos^2\Theta$$

$$R^2=1+r^2$$

$$\sinh u=r\cos\Theta$$

$$e^u=r\cos\Theta+\sqrt{r^2\cos^2\Theta+1}$$

$$=r\cos\Theta+\sqrt{R^2-r^2\sin^2\Theta}$$

$$=\frac{1}{\sqrt{r^2\cos^2\Theta+1}-r\cos\Theta}$$

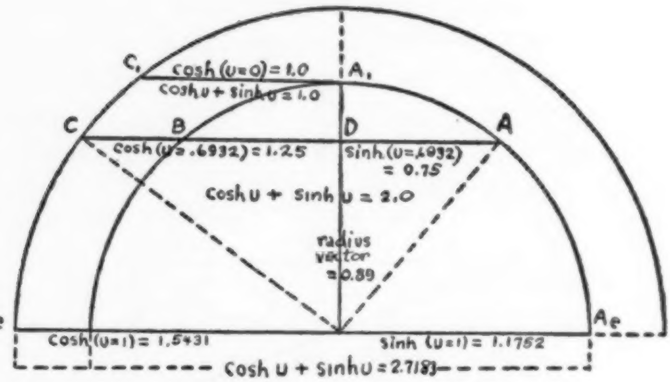
$$=\frac{1}{\sqrt{R^2-r^2\sin^2\theta}-r\cos\theta}$$

$$=\frac{1}{BC}$$

or

$$u=\log\frac{1}{BC}$$

$$R-r\leqq BC\leqq 1,\quad 0\leqq \sinh u\leqq r.$$



THE MIL AS AN ANGULAR UNIT AND ITS IMPORTANCE TO THE ARMY

(Continued from page 400)

In view of the preceding facts it is obvious that many of the students now studying secondary or college mathematics will shortly find knowledge of the mil system highly desirable. However, the mil is mentioned in practically none of the current geometry or trigonometry texts and is seldom included in trigonometry courses at present. Hence, the Sub-Committee on Education for Service of the War Preparedness Committee recommends that teachers of trigonometry add to their courses work involving the mil system along with the usual material on radian measure. The following types of exercises should be included (the reader will no doubt want to add many more types).

1. Convert the angle 36° 10′ 20″ into mils.
2. Convert the angle 22 mils into degrees, minutes and seconds.
3. Convert the angle 0.7 radians into mils.
4. Convert the angle 19 mils into radians.
5. Draw an angle: estimate its value in degrees; radians; mils.
6. How many mils are there in the central angle intercepting an arc of 20 inches on a circle of 25 inches radius?—*(Continued on page 418)*

Humanism and History of Mathematics

Edited by
G. Waldo Dunnington

A History of American Mathematical Journals

By Benjamin F. Finkel
Drury College

(Continued from April issue.)

THE ANALYST
Vol. I No. I.
1874.

Edited and Published by
J. E. Hendricks, A. M.

Des Moines, Iowa
Printed by State Printing Company

From the time of the discontinuance of the *Mathematical Monthly*, September, 1861, until January, 1874, there was, apparently, no journal devoted exclusively to Mathematics, published in the United States. In January, 1874, a new candidate appeared appealing to the favor and support of the Mathematicians of America. This journal was *The Analyst*, edited and published by J. E. Hendricks, A. M., Des Moines, Iowa.

In Vol. I, No. 1, pp. 1, 2, the Editor makes the following Introductory Remarks:

"The present is eminently a period of activity, both physical and mental. Science is daily developing new truths, and thereby increasing the sum of human knowledge. Guided by analysis, the mechanic is daily improving and perfecting labor-saving machinery, thereby augmenting the amount of human happiness; and the astronomer is re-examining his conclusions, and, with the help of new and improved

instruments, correcting his data, thus perfecting our knowledge of the extent and order of the material universe.

As a knowledge of the laws of natural phenomena (and as a consequence the happiness and welfare of mankind) is promoted by community of mind, it is believed that by such an intercourse of thought as this journal is intended to induce, the sum of human happiness will be increased.

The editor is fully aware that no effort on his part alone can make such a publication as this is intended to be, generally interesting to its readers; he only hopes for success in that respect by enlisting as contributors a majority of its readers. He therefore invites all who may feel an interest in its success to contribute to its pages their best thoughts and most valuable conclusions, embodied in brief and concise notes or essays.

As the scientific character of the *Analyst* has not been fully explained by circular, we embrace this opportunity to state that, as its title imports, it is intended to afford a medium for the presentation and analysis of any and all questions of interest or importance in pure or applied Mathematics, embracing especially all new and interesting discoveries in theoretical and practical astronomy, Mechanical philosophy, and engineering.

We are aware of the difficulty of publishing such a periodical as we have above indicated, and of the apparent presumption of attempting it at this place, where we have no prominent institution of learning nor the facilities for printing that might be obtained farther east. Nevertheless, as there seems to be an obvious want of a suitable medium of communication between a large class of investigators and students in science, comprising various grades from the students in our high schools and colleges to the college professor; and, moreover, as we have been encouraged by kind words and promises of assistance from various eminent teachers and professors, which from the contributions received for this, our first number, we have reason to believe will be fully realized, we have determined to venture the publication.

We invite, and expect to obtain the following two classes of persons as readers of our Journal, viz.: 1st, Those who are able and willing to communicate valuable information through the Journal; and, 2nd, Those who desire to increase their stock of knowledge and shall find that desire partly supplied by the Journal.

All who feel an interest in the success of the Journal are respectfully solicited to coöperate with, and assist us, in extending its circulation.

We earnestly solicit contributions for publication from all who desire to promote the interest and usefulness of the Journal. In selecting matter for publication each month, we will present such as we may think most interesting or of greatest utility.

We will publish from three to five mathematical questions in each No., and will endeavor to select such as are believed to be new, or as seem to possess special interest, and will try to grade them so as to suit the different degrees of advancement of our readers. The solutions of mathematical questions will, in general, be published in the second No. succeeding the one in which the questions are published."

The first volume of the *Analyst* was published monthly, the first number appearing, January, 1874. Beginning with the second volume, it was changed to a bi-monthly and remained as such as long as it was published. The price of subscription was $2.00 per year. It was published at Des Moines, Iowa, during the entire time of its publication.

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Note on the Division of Space, by Prof. Hall, p. 140; Correction of an Error, in Barlow's Theory of Numbers, by Artemas Martin, Erie, Pa., pp. 140-142; Note on the Solution of Mr. Holbrook's Question, by the Editor, pp. 142-150; Invention of a New Numerical System, by Ferdinand Eissfeldt, Boston, Mass., pp. 151-152; Solutions of Problems in No. 4, pp. 153-159; Nine problems for solution, pp. 159-160.

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On the Development of the Perturbative Function in Periodic Series, by G. W. Hill, Nyack Turnpike, N. Y., pp. 161-180; Reminicences of Wm. Lenhart, Esq., by Prof. Daniel Kirkwood, pp. 181-182; On Supplying Omissions in a Closed Survey, by Cleveland Abbe, Washington, D. C., pp. 182-184; Solution of a Problem on page 40, No. 2, by Prof. William Woolsey Johnson, Annapolis, Md., p. 184; On the first Principles of the Calculus, by Hiram Cook, C. E., Norwich, Conn., pp. 185-191; Solutions of Problems in No. 5, pp. 191-199; Seven problems for solution, pp. 199-200. On this page the Editor announces, among other things, that he has made definite arrangements for the continuation of the *Analyst* at least for another year and perhaps longer.

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Contents of Vol. III, No. 5.

New Investigation of the Law of Errors of Observation, by Chas. H. Kummell, Assistant U. S. Lake Survey, Detroit, Mich., pp. 135-140;

Solution of the General Equation of the Fifth Degree, by Adolph Von der Schulenburg. "It still moves!"* Translated by Dr. A. B. Nelson, Professor of Mathematics in Center College, Danville, Ky., pp. 141-148.† This sign refers to a note by the translator which is as follows: He says, "My reason for offering this translation for publication is simply the fact that it seems to be unknown among mathematicians in this country that this solution has really been accomplished. As evidence of this generally prevailing impression, I cite the following extract from *Educational Notes and Queries* for Feb., 1875, Edited by Prof. W. D. Henkle, at Salem, Ohio. 'It may not be known to the younger class of mathematicians that the impossibility of resolving algebraically, a general equation of the fifth or higher degrees has been rigorously demonstrated. This was first done by Abel. Wantzel's demonstration which Serret says is simpler than Abel's, may be seen in Serret's *Cours D' Algèbre Supérieure*, 3rd edition, 1866, Vol. 2. Also, the following concluding paragraph from an article by Mr. G. W. Hill, in the *Analyst*, Vol. II, No. 1: 'If we should attempt to treat the general equation of the fifth degree in the preceding manner, we would be led to equations of higher degrees than the fifth, which must be regarded as a strong probable argument for the non-existence of an algebraic expression equivalent to the root of the general equation of this degree.'

"These statements, as far as I am aware, having passed unchallenged for over a year and a half, I conclude that the solution here given to the American Public is not generally known."

TRANSLATOR.

A New Method of Solving Numerical Equations, by Dr. H. Eggers, Milwaukee, Wis., pp. 149-150; Trisection of an Angle, by Dr. Wm. Hillhouse, New Haven, Conn., pp. 151-152; Solution of a Problem in Probabilities, by the Editor, pp. 152-154; Solution of two Problems in Summation of Series, by Prof. D. Trowbridge, Waterburgh, N. Y., pp. 154-155; Solution of Problem 118, by Prof. Johnson, p. 156; Solutions of Problems in No. 4, pp. 157-162; Twelve Problems proposed for Solution, pp. 163-164. Alphabetical Index, two pages following the title page, No. 6, Vol. III.

Contents of Vol. III, No. 6.

New Investigation of the Law of Errors, by Chas. H. Kummell, Assistant U. S. Lake Survey, Detroit, Mich., (continued from page 140), pp. 165-172; Extraction of Roots by Logarithms, by Artemas Martin, Erie, Pa., p. 172; Solution of the General Equation of the

Fifth Degree, Translated by Dr. A. B. Nelson, Danville, Ky., (continued from page 148), pp. 173-178; Reduction of the Problem of Three Bodies, by G. W. Hill, Nyack Turnpike, N. Y., pp. 179-185; Six Original Problems, by Orson Pratt, Sen., Salt Lake City, Utah, pp. 186-188; Solution of Problems in No. 5, pp. 188-191; Five Problems for Solution, pp. 191-192.

Contents of Vol. IV, No. 1.

Singular Solutions of Differential Equations of the First Order, by W. W. Johnson, St. John's College, Annapolis, Md., pp. 1-4; Some Trigonometric Series, by Prof. D. Trowbridge, Waterburgh, N. Y., pp. 4-8; Solution of the General Equation of the Fifth Degree, translated by Dr. A. B. Nelson, Danville, Ky., (continued from p. 173, Vol. III), pp. 9-14; Solution of the Six Original Problems, published in No. 6, Vol. III, by the Proposer, pp. 15-19; Force of Gravity at any Latitude, by R. J. Adcock, Monmouth, Ill., p. 20; Solution of a Problem, by W. E. K. Goudy, A. M., Springfield, O., pp. 20-21; Solutions of Problems in No. 5, Vol. III, pp. 21-25; Solution of Problems in No. 6, Vol. III, pp. 25-30; Ten Problems for Solution, pp. 31-32.

Contents of Vol. IV, No. 2.

On the History of the Method of Least Squares, by Mansfield Merriman, Ph.D., New Haven, Conn., pp. 33-36; On the Rotation of Saturn, by Prof. A. Hall, pp. 36-42; Classification of Plane Curves with Reference to Inversion, by Prof. W. W. Johnson, St. John's College, Annapolis, Md., pp. 42-47; Determination of the Locus of 0, (see fig. on p. 22), by Christine Ladd, Union Springs, N. Y., pp. 47-48; Query, by Prof. A. Hall,—The approximate value of the definite integral

$$\int_0^{\pi/2} \sqrt{\sin\phi}\, d\,\phi \text{ is } 1.198\ldots\ldots:$$

Is there a convenient way of computing this numerical value? p. 48; Solution of the General Equation of the Fifth Degree, translated by Dr. A. B. Nelson, Danville, Ky., (continued from page 14), pp. 49-54; On Problem 132, by Prof. A. Hall, Washington, D. C., p. 55; Remarks by Prof. Orson Pratt, on the Six Propositions, published in the *Analyst*, Vol. III, No. 6, pp. 55-56; Solutions of Problems in No. 1, pp. 57-63; Eleven Problems Proposed for Solution and one Query, pp. 63-64.

Contents of Vol. IV, No. 3.

Demonstration of the Impossibility of Resolving Algebraically, General Equations of a Degree Superior to the Fourth, translated from

Serret's *Cours D'Algèbre Supérieure*, by Alexander Evans, Esq., Elkton, Md., pp. 65-70; Demonstration of a Proposition, by the Editor pp. 71-72; Solution of a Problem, by Prof. David Trowbridge, Waterburgh, N. Y., pp. 73-76; The Rotation of Saturn, by Alexander Evans, Esq., Elkton, Md., pp. 76-77; Remarks on Problem 137, by George Eastwood, pp. 77-79; On Adjustment Formulas, by E. L. DeForest, pp. 87-88; Recent Mathematical Publications, by G. W. Hill, pp. 77-78; Note on a Logarithmic Series, by W. E. Heal, Wheeling, Ind., p. 88; Solutions of Problems in No. 2, pp. 89-95; Eight Problems for Solutions and two Queries, pp. 95-96.

Contents of Vol. IV, No. 4.

Emperical Formula for the Volume of Atmospheric Air, by G. W. Hill, pp. 97-107; On Adjustment Formulas, by E. L. DeForest, M. A., (continued from p. 86), pp. 107-113; Curve of Logarithmic Sines, by Prof. L. G. Barbour, Richmond, Ky., pp. 114-116; Solutions of Problem 155, by Chas. H. Kummell, U. S. Lake Survey, Detroit, Mich., pp. 117-121; Demonstration of Proposition XXV of the First Book of Euclid, by W. E. Heal, Wheeling, Ind., pp. 122-123; Solutions of Problems in No. 3, pp. 123-127; Five Problems for Solution and one Query, pp. 127-128.

Contents of Vol. IV, No. 5.

Short Method of Elliptic Functions, by Levi W. Meech, Hartford, Conn., pp. 129-136; Taylor's Theorem and its Limits, by A. W. Whitcomb, M. D., Sheboygan Falls, Wis., pp. 137-140; Note on the History of the Method of Least Squares, by Mansfield Merriman, Ph.D., New Haven, Conn., pp. 140-143; Pedal Curves, by Prof. W. W. Johnson, Annapolis, Md., pp. 143-146; The Differential Equations of Problem 165, by Prof. A. Hall, Naval Observatory, Washington, D. C., pp. 146-147; Twelve Original Problems, by Pliny Earle Chase, LL.D. Prof. of Philos. in Haverford College, pp. 147-148; Solutions of Problems in No. 3, pp. 149-152; Solutions of Problems in No. 4, pp. 152-158; Seven Problems Proposed for Solution, pp. 158-159; Special Notice:—

As is well known to the readers of the *Analyst*, its publication was undertaken, not with the expectation of gain, but to supply, temporarily, a "medium of communication" between mathematicians and students of mathematics in this country; and it has been, and is, our intention only to continue the publication so long as it will be of service to mathematicians.

Our patronage, so far, has exceeded our anticipations, so that we had no thought of discontinuing the publication at present. But,

through the kindness of Prof. Hall, we have just received (Aug. 11th) a *Prospectus* of *The American Journal of Pure and Applied Mathematics*, to be published at Baltimore; and as most of our subscribers will undoubtedly want the new journal, and many of them may not want to incur the expense of two mathematical journals, we copy, below, the prospectus alluded to, and ask, as a special favor, that all who may desire to discontinue their subscriptions for the *Analyst* at the close of Vol. IV, will notify us before the 1st of October, so that we may be able to announce our intentions for the future in No. 6 of the *Analyst*.

EDITOR OF *Analyst*.

The "Prospectus" referred to above appears under our History of the *American Journal of Mathematics*, which see.

Contents of Vol. IV, No. 6.

Short Method of Elliptic Functions, by Levi W. Meech, A. M., Hartford, Conn., (continued from p. 136), pp. 161-168; Evolute to Curve of Logarithmic Sines, by Prof. L. G. Barbour, Richmond, Ky., pp. 169-171; Quaternions, by Christine Ladd, Union Springs, N. Y., pp. 172-176; Query, by Request.—In Salmon's Higher Plane Curves, 2nd Ed., p. 180, it is stated, "A right angle has the side *GF*, of fixed length, the point *F*, moves along the fixed line *CI*, while the side *GH*, passes through the fixed point *E*, a pencil at the middle point *GF* will describe the Cissoid." How is this demonstrated? p. 176; Note of the Evaluation of Indeterminate Forms, by Prof. W. W. Johnson, Annapolis, Md., pp. 177-178; Answer to Query, page 128, by Prof. D. J. McAdam, Washington, Pa., pp. 178-181; Note on Attraction, by R. J. Adcock, Monmouth, Ill., p. 181; Solution of Prob. 174, by Chas. H. Kummell, (continued from p. 157), pp. 182-183; Note on the Method of Least Squares, by R. J. Adcock, Monmouth, Ill., pp. 183-184; Solutions of Problems in No. 5, pp. 184-190; Note by the Editor on Mr. Adcock's Note on Attraction, page 181, pp. 190-191; Five Problems for Solution, p. 191. Announcement by the Editor: From the many kind and encouraging letters re'cd in response to the special notice published in No. 5, p. 159, in which the Editor of the *Analyst* published, in full, the *Prospectus of the American Journal of Pure and Applied Mathematics* to be published at Baltimore, under the Editorship of William E. Story. More will be said about this Journal later. On page 192 of No. 6, Vol. IV, the Editor announces the continuance of the *Analyst*.

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On the Grouping of Signs of Residuals, by E. L. DeForest, A. M., Watertown, Conn., pp. 1-6; Enumeration of Primes, by Prof. W. W.

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the use of infinitesimals, or limits, by Prof. Ficklin, Columbia, Mo., pp. 177-178; The meteor of August 11, 1878, by Prof. Daniel Kirkwood, pp. 178-180; Quinquisection of the circumference of a circle, by Prof. L. G. Barbour, Richmond, Ky., pp. 180-181; A problem and its solution, by Dr. H. Eggers, Milwaukee, Wis., pp. 181-183; Solution of problems in No. 5, pp. 183-190; Ten problems for solution, pp. 190-191; "An announcement by the Editor, in which he speaks encouragingly of the success of the *Analyst* during the past five years and that while he commenced its publication contrary to the advice of friends, who were nearly unanimous in cautioning him that such a publication at that place, would not be supported; yet, after five years experience, he was pleased to say that the support received had exceeded his expectations and that neither the locality of its publication nor the obscurity of its editor had prevented the *Analyst* from receiving the patronage and support of many of the best Mathematicians, Educators, and public institutions in America, he therefore, embraced the opportunity at that time to inform his readers that the *Analyst* will be continued as long as his health permits and the interest of subscribers in its continuance is manifested by suitable contributions for publication, p. 192." Writer. Alphabetical Index, two last pages of No. 6, Vol. V.

THE MIL AS AN ANGULAR UNIT AND ITS IMPORTANCE TO THE ARMY

(*Continued from page 402*)

7. What length of arc at 2000 yards will 3 mils intercept?
8. A circular target at 5000 yards subtends an angle of 2 mils, at the edge. What is the diameter of the target?
9. From the position of an observer, an automobile 20 feet in length at right angles to the line of sight subtends an angle of 2 mils. What is its distance from the observer?

Problems involving the solution of triangles where the angles are measured in mils, should also be included.

The student should be given some field practice in estimating angles in mils; and he should be encouraged to work many problems involving the principles encountered in Exs. 6, 7, 8, and 9, approximately and without the use of pencil and paper.

Teachers at the secondary level should note that problems like the preceding Examples 1 through 9 could be included in the course in plane geometry if radian measure and related principles are discussed.

For the convenience of the reader a few conversion factors are listed below:

90 degrees = 1600 mils	1 radian = 57° 17′ 45″
1 degree = 17.77778 mils	1 radian = 1018.6 mils
1 minute = 0.296296 mils	1 degree = 0.0174533 radians
1 mil = 0.05625 degrees	1 minute = 0.0002909 radians
1 mil = 3.37500 minutes	1 mil = 0.0009817 radians

The Teacher's Department

Edited by
JOSEPH SEIDLIN and JAMES McGIFFERT

Mathematics and the Engineering Curriculum

By DWIGHT F. GUNDER
Colorado State College A. & M.

It is the purpose of this paper to discuss one of the most important contributions of mathematics to the engineering student. This contribution can be stated rather generally as the development of the method of deductive reasoning; the reasoning from a set of fundamental laws or assumptions to their useful conclusions. Almost all that mathematics should contribute to the engineer in the way of mechanical skills, ability to attack original problems, and ability to think in terms of symbols, is based predominantly upon this one method of reasoning. It is the failure to grasp this thought process at an early stage in the educational program and the failure to follow it throughout that program that makes mathematics and associated subjects difficult and intangible to many students.*

The lack of mastery of this thought process on the part of the student becomes apparent at once to the instructor in algebra when he finds, for example, that the student violates fundamental principles by multiplying one member of an equation by a quantity without at the same time multiplying the other member by the same quantity. As an illustration, the erroneous simplification that $x/2-y/3=1$ is equivalent to $3x-2y=1$, is usually the direct result of the failure of the student to reduce his thinking to the very few operations allowable upon an equation. In this illustration the student wishes to eliminate the fractions. These fractions themselves indicate division, hence he

*Two things should be made clear at this point. In the first place the so-called fundamental laws and axioms may not be true. In the second place certainly the deductive method is not the only method of reasoning. It is not my intention to discuss either of these points, rather I am assuming that these fundamental laws and axioms and this method of reasoning have borne fruit generously to the engineer and that they are worthy of our careful consideration.

must perform the inverse operation, multiplication. Since he is dealing with an equation the only suitable allowable operation is multiplication of both members by the same quantity, in this case by 6 the least common multiple of the given denominators.

The same type of difficulty, that is failure to grasp the underlying principles, is at once obvious to the instructor in physics or mechanics when he finds the student attacking a problem by going back through the preceding material and trying every formula which he finds that might apply to his problem, then substituting the known data into each of these formulas in hopes that some one will yield the correct solution to the problem. Here it is the failure on the part of the student to realize that usually his problems can best be solved by the use of the fundamental definitions, laws and principles, and a *very few* resulting formulas, the latter in almost all cases being carefully derived and certainly in all cases being thoroughly understood. In general it would seem that the fewer formulas used in mathematics, physics, and mechanics, the better for the development of the power of analysis and understanding by the student.

Granting then that it is desirable that the student be taught to reason out the solutions to his problems on the basis of certain fundamental laws or assumptions, how can the instructors, particularly in mathematics and engineering, cooperate in developing this reasoning power in the student?

Certainly the first and basic obligation lies with the instructor in mathematics. Upon him falls the duty of developing the simple underlying principles of manipulation with quantities and equations. These basic principles are well stated and constantly referred to in the development of elementary geometry. They are also well stated in elementary arithmetic and algebra but for some reason are not so often directly referred to in these courses. Perhaps it would be well to state briefly the more commonly used principles in algebra and geometry. They are: (1) only like quantities may be added (subtracted), (2) the only operation which does not change the value of a fraction is multiplying (dividing) its numerator and denominator by the same nonzero quantity, (3) the same or equal quantities may be added to or multiplied by the same or equal quantities to give equal results, and equal quantities may be raised to equal powers with equal results.* Many of these principles are already familiar as used in arithmetic; the others are fairly readily understood. These principles taken with the definitions of various algebraic quantities are the tools with

*There are of course certain restrictions upon these operations relative to operations with zero, the notion of principle roots, et cetera, which although essential in a good presentation to the student are omitted here for the sake of brevity.

which the instructor and student in algebra enlarge upon the work of arithmetic.

At first the method of instruction is fairly standard with the principles and laws being carefully introduced at the beginning of the course. Beyond this point, however, there is a more or less sharp division among instructors. Almost before the student has begun to understand what he is doing, one group of teachers proceeds to develop certain special devices which although correct in themselves apply only to particular cases. These devices or, "rules" make teaching temporarily easier by substituting rule for reason in the solving of problems. However, the student soon becomes lost in a maze of catch phrases, such as "cross multiply", "invert and multiply", "cancel out", "clear of fractions", and many others until the rules finally become useless. He has forgotten the original principles and his teachers say, "he just doesn't have a mathematical mind." The above expressions are correct when their limitations are known and they are properly applied, but when overworked they tend both to smother the student's understanding and to lead him into inexact expression of his ideas.

A second group of teachers adheres more closely to the original principles and develops, perhaps more slowly at first but more soundly, a group of students who proceed understandingly through the solutions of their problems. It is this second group of teachers who, in my estimation, will develop the students best trained to apply their mathematics to all fields of study and in particular to the field of engineering.

Although limited space forbids either a complete discussion of algebra or similar discussions for the other mathematics courses, the same process of developing complete knowledge of fundamentals can and should be carried through. For example, in the differential calculus a great deal more power and a great deal less confusion results from limiting the number of formulas and emphasizing by more frequent use of implicit differentitation the operations already learned in algebra and trigonometry.

In line with this I should like to say that at present there seems to be a definite trend especially in the newer mathematics texts toward limiting the contents to basic principles in an effort to produce a more thorough understanding of these principles and their applications. This is a step in the direction I am recommending, for it results in a reduction in the bulk of the textbook and leaves the student with a simpler, clearer statement of the principles and a more thorough understanding of the method of attack. He spends more time digesting and assimilating and less in learning devices which though useful and

beautiful to the mathematician may be far from practical for the engineer.

So far I have mentioned only the part of the mathematics instructor in the development of the student. What then is the part to be played in this development by the instructor in engineering? As I see it from a background of mathematics, physics and engineering, the instructors in these last two groups need only to continue the method of developing fundamental ideas and building upon these firm foundations. Here as in mathematics I shall discuss only one of the courses in engineering. I shall choose statics as the one for consideration since it is normally one of the first courses met by the engineering student in his own division where theory is presented and developed. Furthermore this course is almost an ideal one in which to apply the method of developing from fundamental principles. Here the principles are relatively few, being really only three in number, the principle of resolution of forces and the principles that for equilibrium the algebraic sum of the forces in any direction and the moments about any line or point must be zero. Here then is the golden opportunity to get the student's feet firmly planted on the ground of understanding. If he can thoroughly master just these three ideas he has a sound basis for all the rest of his mechanics. It is the duty of the mechanics instructor to see that he masters them by constant use and application in as many different problems as possible. Unless the student does master this course he can only expect to be hopelessly lost in those that follow.

In conclusion, the instructors in mathematics and physics must cease trying to teach their courses as though the students were planning to major in these fields and remember that they are not trying to make expert mathematicians and physicists of these students but rather that they are trying to give them an understanding of those basic principles and skills of the courses which will do most toward developing them for their ensuing engineering work. At the same time the instructors in engineering courses must develop their courses on sound fundamentals. Furthermore, in their use of mathematics they should proceed with the same understanding and application of the basic principles of mathematics as would an instructor in mathematics. Certainly the engineering instructor may be less expert in the manipulation of these fundamental principles but he should use them with full confidence and understanding.

If then the teachers of mathematics, physics and engineering will modify their courses slightly and cooperate in the formulation of a sound and fairly uniform general attack on the teaching problem, the result should be a coordinated course of study which I believe would produce the maximum result in student development.

Mathematical World News

Edited by
L. J. Adams

Professor F. H. Sibley, dean of the college of engineering in the University of Nevada, died on April 2, 1941.

Dr. A. H. Clifford, Massachusetts Institute of Technology, has been promoted to the rank of assistant professor of mathematics.

Dr. Vannevar Bush is chairman of the National Defense Research Council. He has published in the *Science* weekly the names of the scientists and engineers who have accepted definite appointments to work with his committee. These lists are in the October 31, 1940 and April 11, 1941 issues of *Science.*

The annual summer meeting of the National Council of Teachers of Mathematics will be held in Boston, Massachusetts, June 29-July 3, 1941.

The Boletin Matematico, Argentina, is now in its thirteenth year. It is at the present time the unique Spanish publication not only in Latin America but all over the world. The issues include articles, bibliographical information, notes, miscellany and a problem department.

Mathematical Reviews, now appearing monthly, is sponsored by the American Mathematical Society, Mathematical Association of America, Academia Nacional de Ciencias Exactas, Fisicas y Naturales de Lima, London Mathematical Society, and others. Its abstracts and reviews of current mathematical literature are very useful to research workers.

The American Philosophical Society has set aside $10,000 from its budget to use as a gift to the Royal Society of London to be used in aid of science and learning.

The Sylvester Medal has been awarded to Professor G. H. Hardy, the noted English mathematician. Professor Hardy has written some 300 research papers and several of the Cambridge Mathematical Tracts. Among his many achievements some of the most important have been in connection with Tauberian theorems, Riemann Zeta-functions, and the theory of numbers.

Regional Governors of the Mathematical Association of America include Professors F. W. Owens, E. J. McShane, S. T. Sanders, W. C. Krathwohl, Cornelius Gouwens, H. J. Ettlinger, H. M. Bacon.

Professor L. R. Ford has been elected editor-in-chief of the *American Mathematical Monthly* for a five-year term beginning with the issue for January, 1942.

Professor R. W. Brink has been elected President of the Mathematical Association of America for the academic year 1941-1942.

Students majoring in mathematics and also graduates of engineering colleges might be interested in information concerning "navigator" training in the Air Reserve. Such information is contained in the *American Mathematical Monthly.*

The Wisconsin Section of the Mathematics Association of America was scheduled to meet at Beloit College, on May 3, 1941. A morning session and an afternoon session were planned. At the morning session the following program was scheduled:

1. *On Descartes' Rule of Signs and Its Extensions.* Professor Morris Marden, University of Wisconsin Extension Division.
2. *Applications of Mathematics in Radio.* Professor R. D. Spangler, LaCrosse State Teachers College.
3. *Applications of Mathematics to Engineering.* Mr. Alan D. Freas, U. S. Forest Products Laboratory, Madison.

At the afternoon session a panel discussion was planned, with Professor H. P. Evans, University of Wisconsin, presiding:

1. *Contents of Secondary Mathematics.* Miss Irene Eldredge, West Division High School, Milwaukee.
2. *Secondary School Preparation for College Mathematics.* Mr. Walter W. Hart, Kenilworth, Illinois.
3. *Curriculum Problems in a High School Located in a University Community.* Mr. Ralph O. Christopherson, Assistant Principal, West Side High School, Madison.

Volumes could be written on boners committed in the calculus. The first volume (probably entitled, *The New Differentiation*) should contain this example from the paper of a student, as we humorously call them, of the calculus:

$$D_x\arcsin(x^2-3x) = \arcsin D_x(x^2-3x) + (x^2-3x)D_x\arcsin$$

$$\arcsin(2x-3) + (x^2-3x)\cdot\frac{1}{\sqrt{1-\cos^2}}\cdot$$

Problem Department

Edited by
ROBERT C. YATES and EMORY P. STARKE

This department solicits the proposal and solution of problems by its readers, whether subscribers or not. Problems leading to new results and opening new fields of interest are especially desired and, naturally, will be given preference over those to be found in ordinary textbooks. The contributor is asked to supply with his proposals any information that will assist the editors. It is desirable that manuscript be typewritten with double spacing. Send all communications to ROBERT C. YATES, Mathematics, University, Louisiana.

SOLUTIONS

Late Solutions: Nos. 381, 383, 384 by *C. W. Trigg.*

No. 348. Proposed by *Alfred Mœssner*, Nurnberg, Germany.

Find the general solution in integers for the equations

$$A_1+A_2+A_3=B_1+B_2+B_3=X^2,$$

$$A_1\cdot A_2\cdot A_3=B_1\cdot B_2\cdot B_3=Y^2.$$

(For example: $A_1=9$, $A_2=A_3=20$, $B_1=B_2=12$, $B_3=25$, $X=7$, $Y=60$.)

Editor's Note: Noting the form of the Proposer's example, we may set $A_i=u^2, vw, vw$; $B_i=v^2, uw, uw$. Thus

$$(1) \qquad y=uvw, \qquad x^2=u^2+2vw=v^2+2uw.$$

From this result, we have

$$u^2-v^2-2uw+2vw=0 \quad \text{or} \quad (u-v)(u+v-2w=0.$$

Since $u=v$ makes A_i identical with B_i, set

$$w=\tfrac{1}{2}(u+v), \quad x^2=u^2+uv+v^2.$$

The general solution in positive integers of this last equation is easily found as

$$u=k(2mn-n^2), \quad v=k(n^2-m^2), \quad x=k(m^2-mn+n^2),$$

where m, n are relatively prime, $m<n<2m$, and k is an integer or a third of an integer according as 3 is not or is a factor of $mn+1$.

Further, if $u+v$ is odd, we choose k even, so that w is integral. Thus are obtained infinitely many solutions of the proposed equations: the Proposer's example results from $m=2$, $n=3$, $k=1$.

This is evidently not general. Assumed relations other than (1) lead to other results: for instance

$$2uvw=y, \quad 4uv+uw+vw=uv+2vw+2uw=x^2;$$

$$3uvw=y, \quad 3u^2+3vw+vw=3v^2+3uw+uw=x^2.$$

Examples are, respectively: $A_i=42, 294, 448$; $B_i=84, 112, 588$; $x=28$, $y=2352$. $A_i=48, 429, 1287$; $B_i=78, 234, 1452$; $x=42$, $y=5148$.

No. 357. Proposed by *Cyril A. Nelson*, New Jersey College for Women.

Show how to arrange eight couples at bridge in such a fashion that in seven hands each man plays once with and once against each woman except his wife; each man plays once against each other man, and each woman plays once against each other woman.

Show that an analogous arrangement for six couples is not possible. Can a general statement be made concerning $2n$ couples?

Editor's Note: The problem is essentially the same as No. E 208, the *American Mathematical Monthly* for August, 1938, p. 479, in which a solution of the present first paragraph is printed. To show the impossibility of a similar arrangement of six couples, arrange first the men, any three of whom we designate by A, B, C. Let round 1 be that in which A plays B, and let D designate C's opponent in this round. Let round 2 be that in which A plays C. The men at the third table of round 1 cannot again play together; one of them (say E) plays B, and the other, F, plays D. Then the other rounds are uniquely determined and we have: *AB, CD, EF*; *AC, BE, DF*; *AD, BF, CE*; *AE, BD, CF*; *AF, BC, DE*. There is thus a unique arrangement (except for order of the rounds) if we do not designate the men, D, E, F, until they are placed. (If we had not the privilege of such designation—as we shall not have below in the case of the women—there would have been six arrangements, corresponding to the permutations of D, E, F.)

With A's wife indicated by a, etc., let us arrange the women independently; after which try to combine the arrangements of men and women at the tables in such a fashion that no man and his wife are together at the same table. The following observations are easily verified: if, for a single round, the men and women are paired alike (e. g. *AB, CD, EF*; *ab, cd, ef*) they can be combined in two ways (*ABcd, CDef, EFab*; or *ABef, CDab, EFcd*); if they are alike in just one pair

(e. g. *AB, CD, EF*; *ac, bd, ef*) there is no way of combining; if no pairs are alike (e. g. *AB, CD, EF*; *ac, bf, ed*) there is one way of combining (*ABed, CDbf, EFac*). Thus each round in each of the six arrangements of the women can be combined with only certain rounds in the arrangement of men. If the rounds be reordered in any of the ways which conform to the results just obtained, it will be found that *A* is required to play at the same table as some certain woman at least three times, except when the arrangement of women is identical with the arrangement of men and also the rounds are taken in the same order. Finally there appear just four ways in which each man can play at a table once with each other man and twice with each woman except his wife. One such arrangement is

Round 1	Round 2	Round 3	Round 4	Round 5
ABef	*ACdf*	*ADce*	*AEbd*	*AFbc*
CDab	*BEac*	*BFad*	*BDcf*	*BCde*
EFcd	*DFbe*	*CEbf*	*CFae*	*DEaf*.

A moment's experiment with this and the other three arrangements will show the impossibility of the final requirement that each woman be each man's partner once and only once.

No general statement about $2n$ couples has been suggested.

No. 375. Proposed by *H. S. Grant*, Rutgers University.

What are the necessary and sufficient conditions that the polynomial

$$y=\sum_{i=0}^{n} a_i x^i, \quad a_i \text{ real}, \quad a_n \neq 0, \quad n \geqq 3,$$

be reducible to $Y=a_nX^n$ by a translation of axes. Where is the new origin?

Solution by *C. W. Trigg*, Los Angeles City College.

With $Y=y-k$ and $X=x-h$, $Y=a_nX^n$ becomes

$$y-k=a_n(x-h)^n \quad \text{or}$$

$$(1) \qquad y=a_n[x^n-{}_nC_1x^{n-1}h+{}_nC_2x^{n-2}h^2-\cdots+(-1)^nh^n]+k.$$

Comparison of coefficients of like powers in (1) and in

$$y=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_0 \quad \text{gives}$$

$$a_0=k+(-1)^na_nh^n \quad \text{and} \quad a_i=(-1)^{n-i}{}_nC_ia_nh^{n-i}, \quad i=1, 2, \cdots, n.$$

A necessary condition that the given polynomial be reducible to $Y=a_nX^n$ by a translation of axes is given by

$$a_i/a_{i+1}=-{}_nC_ih/{}_nC_{i+1}=-(i+1)h/(n-i),$$

which states that $(n-i)a_i/(i+1)a_{i+1}$ is a constant for $0<i<n$. The new origin will be (h,k) where

$$h=-a_{n-1}/na_n, \quad k=a_0-(-1)^na_nh^n.$$

Since the condition given above determines uniquely every coefficient (except a_0 and a_n) such that (1) is satisfied, it is sufficient as well as necessary.

No. 386. Proposed by *N. A. Court*, University of Oklahoma.

In a given plane to find a point such that its harmonic plane for a given tetrahedron shall pass through the harmonic pole of the given plane for the same tetrahedron.

Solution by *W. T. Short*, Oklahoma Baptist University.

Let the given tetrahedron be $A\ B\ C\ D$ with coordinates (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1).

Then if
$$\frac{x_1}{e_1}+\frac{x_2}{e_2}+\frac{x_3}{e_3}+\frac{x_4}{e_4}=0$$

is the equation of the given plane, it is well known or easily proven that the harmonic pole of the plane with respect to the tetrahedron is the point M with coordinates (e_1,e_2,e_3,e_4).

Let the edges a, b, c, a', b', c' cut the plane in the respective points x, y, z, u, v, w. Since a is the line $x_1=x_4=0$ it has coordinates $(0, e_2, -e_3, 0)$. Likewise the coordinates of the other five points in order are: $(-e_1, 0, e_3, 0)$, $(-e_1, e_2, 0, 0)$, $(e_1, 0, 0, -e_4)$, $(0, e_2, 0, -e_4)$, $(0, 0, e_3, -e_4)$.

Any point on $(u\ x)$ may be represented by $(e_1, -re_2, re_3, -e_4)$ where r is a parameter. The polar plane of this variable point is

$$\frac{x_1}{e_1}-\frac{x_2}{re_2}+\frac{x_3}{re_3}-\frac{x_4}{e_4}=0\quad.$$

But this is satisfied by the coordinates of M: (e_1, e_2, e_3, e_4). The same is true for the line $(v\ y)$: $(-se_1, e_2, se_3, -e_4)$ and for the line $(w\ z)$: $(te_1, -te_2, e_3, -e_4)$.

Thus, any point on the sides of the diagonal triangle of the complete quadrilateral formed by the intersection of the given plane and

the given tetrahedron is a point such that its polar plane passes through the point which is the pole of the plane of this quadrilateral.

Also solved by *P. D. Thomas* and the *Proposer.*

No. 387. Proposed by *W. V. Parker*, Louisiana State University.

If the curve $y=x^4+ax^3+bx^2+cx+d$ has two points of inflection and intercepts segments of lengths d_1, d_2, d_3 in order on the line joining them show that

$$2d_1=2d_3=(\sqrt{5}-1)d_2.$$

Solution by *C. W. Trigg*, Los Angeles City College.

Without loss of generality the axes may be translated so that the origin coincides with a point of inflection, whereupon the equation of the quartic assumes the form,

$$Y=X^4+AX^3+CX, \quad (A\neq 0). \tag{1}$$

$$Y''=12X^2+6AX.$$

Hence the points of inflection are (0,0) and

$$(-A/2, \quad -A^4/16, \quad -AC/2).$$

The line joining these points is $Y=(A^3/8+C)X$. The abscissas of the intersections of this line and the quartic are roots of the equation, $X^4+AX^3-A^3X/8=0$.

$$X(X+A/2)(X^2+AX/2-A^2/4)=0.$$

So the abscissas of the points which are not inflection points are $A(\pm\sqrt{5}-1)/4$. The four abscissas may be ordered as $A(-\sqrt{5}-1)/4$, $-A/2$, 0, and $A(\sqrt{5}-1)/4$.

Now the lengths of the intercepted segments will be proportional to the lengths of their projections on the $X-$axis, so

$$d_1 : d_2 : d_3 :: (\sqrt{5}-1) : 2 : (\sqrt{5}-1).$$

Hence
$$2d_1=2d_3=(\sqrt{5}-1)d_2.$$

Also solved by *Munson H. Pardee* and the *Proposer.*

No. 389. Proposed by *E. P. Starke*, Rutgers University.

Tangents to the parabola $y^2=4ax$ at any two points A, B meet in a point whose ordinate is the arithmetic mean of the ordinates of

A and B, and whose abscissa is a geometric mean of the abscissas of A and B.

Solution by *W. Raymond Crosier*, Student, Colgate University.

The tangents to the parabola at any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ are given by the equations

$$y_1y=2a(x+x_1), \quad y_2y=2a(x+x_2),$$

whose simultaneous solution gives the coordinates of the point of intersection as

$$x=(x_1y_2-x_2y_1)/(y_1-y_2), \quad y=2a(x_1-x_2)/(y_1-y_2).$$

When these expressions are transformed into functions of x only and of y only, respectively, we have

$$x=\pm(x_1x_2)^{\frac{1}{2}}, \quad y=\tfrac{1}{2}(y_1+y_2),$$

as required.

Also solved by *W. N. Huff, D. L. MacKay, Charles Templeton* and *P. D. Thomas* who found the problem in C. Smith, *Conic Sections*, p. 114.

Editor's Note: The abscissa of the point of intersection is the positive or negative geometric mean according as ay_1y_2 is positive or negative.

No. 391. Proposed by *D. L. MacKay*, Evander Childs High School, New York.

According to Louis C. Karpinski, the following theorem, ascribed to Archimedes by Albiruni (c. 1000 A. D.) was recently brought to light in connection with an Arabic work on Trigonometry and is now believed to have been the basis of Greek Trigonometry before Ptolemy (150 A. D.).

Given cords AB and BC, $AB>BC$, prove that if M is the midpoint of the arc ABC, then MN, perpendicular to AB at N, bisects the broken line ABC.

Solution by *R. V. Sweeney*, Student, Colgate University.

Extend AB to R, so that $NR=AN$. Complete the triangle AMR which is isosceles. Next, complete the triangle MCR in which $MC=MR$. Angle BCM equals angle BAM and is therefore equal to the other base angle of the triangle AMR. Accordingly, triangle CRB is isosceles. It follows that $RB=CB$ and the theorem is evident.

Also solved by *Walter B. Clarke, C. W. Trigg*, and the *Proposer*.

PROPOSALS

No. 415. Proposed by *Richard L. Seidenberg*, Student, Colgate University.

A design is to be made showing the "Red Cross" inscribed to a given circle. What must be the dimensions of the cross so that the area is a maximum. Give a geometric construction.

No. 416. Proposed by *William E. Byrne*, Virginia Military Institute.

Let $$f(x)=x,\quad 0\leqq x\leqq 1,\qquad f(x)=2-x,\quad 1\leqq x\leqq 2,$$
$$f(x+2)=f(x).$$

Find the absolute maximum (maximum maximorum) of

$$x^{-1}\int_0^x f(t)dt,\qquad x>0,$$

and the absolute minimum of

$$x^{-1}\int_0^x f(t)dt,\qquad x>1.$$

No. 417. Proposed by *Paul D. Thomas*, Student, Oklahoma University.

Show that the family of curves which satisfies $(ds/dx)^2=R$, where ds is the element of arc and R the radius of curvature is a two parameter family of catenaries.

No. 418. Proposed by *G. W. Wishard*, Norwood, Ohio.

The sum of the cubes of the first n natural numbers is the sum of the first P odd natural numbers: the sum of the cubes of the first n odd natural numbers is the sum of the first Q natural numbers. Prove these statements for every n, and determine P and Q.

No. 419. Proposed by *Paul D. Thomas*, Student, Oklahoma University.

A variable line meets two fixed skew lines in points P and Q such that angle PRQ is a right angle, where R is the midpoint of the common perpendicular to the skew lines. (a) Find the locus of the line PQ. (b) Find the locus of the midpoint of PQ.

No. 420. Proposed by *Nelson Robinson*, Louisiana State University.

Let

$$A_{ij}^{(n)}=\sum_k(-1)^k\frac{i!j!(n-k)!}{n!(i-k)!(j-k)!k!}\qquad k=0,\cdots,j;\quad k\leqq i\leqq j,\quad i,j=0,\cdots,n,$$

$$B_{ij}^{(n)}=\frac{(n-i)!(n-j)!}{(n-i-j)!n!}\qquad n\geqq i+j,$$

$$B_{ij}^{(n)}=0,\qquad i\leqq n\leqq i+j.$$

Prove $\quad A_{ij}^{(n)}=B_{ij}^{(n)}.$

No. 421. Proposed by *Walter B. Clarke*, San Jose, California.

Through a point P in the plane of a given triangle lines are drawn bisecting the area of the triangle. Discuss the location of points P for which there are one, two, or three bisecting lines.

No. 422. Proposed by *C. N. Mills*, I. S. N. U., Normal, Illinois.

Given any five points, no three of which are collinear, we can set up directly a sixth order determinant which gives the equation of the conic determined by the points. Is there a dual theorem for the envelope of five lines, no three of which are concurrent? That is, can a sixth order determinant be set up directly from the coefficients in the equations of the five lines? Do not find the points of contact.

The Reed College Bulletin for November, 1940 is entitled *A Report on the Mathematics Teaching Seminar*. The report is by Frank Loxley Griffin, Professor of Mathematics at Reed College, in collaboration with Harry E. Goheen, L. Louise Johnson, Robert A. Rosenbaum and Henry Scheffé, teaching fellows in mathematics at Reed College. The teaching seminar to which the report refers was conducted during the academic year 1939-40. The report is issued in the form of a printed booklet, some 34 pages in length. Persons interested in such matters as the teaching of mathematics, mathematics in the curriculum, unified courses in mathematics, needs for mathematics in other fields of knowledge, etc. (all at the college level) would be interested in this publication.

—Reported by L. J. ADAMS.

Bibliography and Reviews

Edited by
H. A. SIMMONS and JOHN W. CELL

An Introduction to Differential Geometry. By Luther Pfahler Eisenhart. Princeton University Press, Princeton, 1940. x+304 pages. $3.50.

Several books on differential geometry have been published in recent years. Some of them have differed from the older classics on the subject in the use of vector analysis, matrix theory, and tensor calculus. Dean Eisenhart's contribution is a classic treatment written in the modern tensor notation. It serves both as a treatise on differential geometry and an introduction to tensor analysis.

Chapter I deals with the theory of space curves. Chapter II introduces the reader to the theory of tensors in connection with transformations of coordinates and the fundamental quadratic form. Special attention is paid to the integrability conditions for systems of partial differential equations. Chapter III treats the classical intrinsic geometry of surfaces (first fundamental quadratic form). Chapter IV discusses the second fundamental quadratic form (surfaces viewed from the enveloping space) and its applications.

The exposition is centered on the theory of curves and surfaces in three-dimensional space, but in such a way that generalization to higher dimensional spaces is immediate. Left-hand axes are used.

The typography and illustrations are excellent. Only one misprint was observed (equation 12.13, page 58).

This book should be required reading for all those who wish to familiarize themselves with the tensor notation and its applications to differential geometry.

Virginia Military Institute. W. E. BYRNE.

Mathematics in General Education. A Report of the Committee on the Function of Mathematics in General Education, for the Commission on Secondary School Curriculum and for the Progressive Education Association. D. Appleton-Century Company, New York, 1940. xiv+423 pages.

Two important and greatly needed reports on secondary school mathematics appeared last year. Because one, *The Place of Mathematics in Secondary Education,* was prepared by a Joint Commission sponsored by two mathematics organizations, the Association and the National Council; and the other, the subject of this review, was prepared by a Committee of a Commission established by the Progressive Education Association, the two are in many ways supplementary. The high school mathematics teacher is fortunate in having both points of view so adequately presented in the same year; and, in the light of all that is being carelessly said on the place of mathematics in the high school curriculum, he should be particularly grateful for the report of a Committee sponsored by the Progressive Education Association. In the seventeen years which have elapsed since the publication of the *Reorganization of Mathematics in Secondary Education,* a report of the National Committee on Mathematical Re-

quirements under the auspices of the Mathematical Association of America, many aspects of secondary education have been greatly changed.

The Commission on Secondary School Curriculum was established in 1932 and subsequently various committees were determined. The Reports of the Committees dealing with *Language*, *Science*, the *Visual Arts*, and the *Social Studies* have already been published. The Committee on the Function of Mathematics in General Education included those whose special field is education as well as college and high school mathematics teachers.

The content of this Report is divided into four parts. The first, covering less than one seventh of the pages, includes a discussion of the purpose of general education with emphasis on certain "major ideals of democracy", and an analysis of the role of mathematics in achieving the outlined purpose. The reviewer was gratified by the attention given to the possible contribution of the mathematics teacher to the students' growth in such desirable qualities as social sensitivity, esthetic appreciation, tolerance, co-operativeness, self-direction, and creativeness.

Part II, occupying about 45% of the whole Report, *Major Understandings Growing out of Mathematical Experience*, is, in the reviewer's opinion, the most valuable Part. For the college teacher who does not have time to read the entire Report, Part II is strongly recommended. A chapter is devoted to each of seven concepts which the Committee believes to be "basic to problem-solving, crucial in democracy, pervasive in mathematics". These concepts are *Formulation and Solution*, *Data*, *Approximation*, *Function*, *Operation*, *Proof*, and *Symbolism*.

Although this Report gives the function concept a place of less prominence than it had in the 1923 report, the Committee nevertheless recognizes the importance of the function concept in modern mathematics. Two goals recommended in the section on the *Formulation and Solution* concept are the ability to "formulate and solve" problems and the ability to "recognize the characteristics of promising formulations and the nature of acceptable solutions". The discussion of the latter goal is particularly good. In this chapter mathematics teachers and texts are criticized for presenting problems so idealized that the student acquires little experience in formulation, as a part of solution.

The reviewer believes that most mathematics courses give too little attention to the concept of data as discussed in the Report, and that there is a place in general education for a study of means of collecting and organizing data, and, more important, for directing study toward an understanding of the "nature of Data". After a discussion of approximations in measurement and in theory, seventeen pages of the chapter on *Approximation* are devoted to "statistical concepts". The reader may conclude that the Committee deems it advisable to teach tthe computation and interpretation of measures of central tendency, dispersion, trend, and correlation in the secondary school.

Recognizing overemphasis on techniques as one of the weaknesses of present-day instruction, the Committee looks for correction in "stressing concepts basic to operation" and suggests possibilities for such emphasis from arithmetic through the calculus. In the chapter on *Operation* the Committee expresses its opinion on drill and on the importance of accuracy. The concept of *Proof* is given a broad interpretation. In this chapter attention is called to the fact that secondary school mathematics has neglected opportunities for the "cultivation of the ability to generalize" as well as for "the study of values and dangers of induction". Teaching of necessary and sufficient conditions is also recommended. Certain plausible means of effecting the Committee's recommendation (that the "uses of symbols in non-mathematical language" be brought

into the study of the use of symbols in mathematics) are included in the chapter on *Symbolism.*

The secondary school teacher should find much of value in the well written section, Part III, on the *Development and Nature of Mathematics.* Many readers of the NATIONAL MATHEMATICS MAGAZINE will probably agree with the view that there is a place in general education for knowledge of the history of mathematics and an understanding of the nature of mathematics.

The reviewer lacks sympathy with the importance attached to Part IV, *Understanding the Student and Evaluating His Growth,* largely because he believes that the content will be familiar to the well-trained teacher through his preparation and his teaching experience. More than thirty pages are devoted to a detailed illustrative discussion of a boy who is a good student in mathematics.

Evaluation of Student Achievement, the title of Chapter XIII, is given an important place today in most discussions of problems of education. The sample evaluation techniques are worthwhile whenever difficulty in the preparation of such evaluation instruments does not discourage the teacher

Those who feel that the Report on the whole is not specific enough may find some satisfaction in the two appendices. One rather long "source unit" and shorter activities illustrative of each of the seven concepts are included.

A viewpoint emphasized by the Committee with which many readers may disagree is that "curriculum sequences should be planned primarily on the basis of concrete problems encountered in meeting educational needs". Others may doubt that secondary school mathematics should be so definitely built around problem-solving. But even in view of these criticisms and others, which, of course, will be made, the reviewer feels that many parts of the Report must be valued and appreciated and that those high school teachers of mathematics who study this report carefully and weigh its recommendations in the light of their own training and experience will, as a result, make a much more significant contribution to general education.

Southern Illinois State Normal University. JOHN R. MAYOR.

The Development of Mathematics. By E. T. Bell. McGraw-Hill Book Company, New York, 1940. XIII+583 pages.

In this book the author carries through a project which must have been in his mind when he wrote his "best seller", *Men of Mathematics.* There we were told what the great mathematicians had contributed to the progress of their science; here mathematics itself has the center of the stage, and its development is traced from the earliest times to the year 1940.

In the preface, this book is described as "a broad account of the development of mathematics, with particular reference to the main concepts and methods that have, in some measure, survived." It is "not a history of the traditional kind. ...Only the most ingenious instructor could set an examination on this book." It tells "why certain things continue to interest mathematicians, technologists, and scientists, while others are ignored or dismissed as being no longer vital."

The author hopes that it may be read with profit by those who plan to end their mathematical education with the calculus or even earlier. For those who are going further, the book is to be a guide to what are the live issues in mathematics, and a deterrent to the sale of mathematically dead horses to gullible pupils by conscienceless or misinformed masters. To put out such a "form sheet" requires courage, as well as the best of information. The author is not afraid to make a judgment where he feels rea-

sonably sure of his ground, or to reserve a decision when caution is in order. The sources of his information are indicated in the sixteen pages of notes at the end of the volume, which might have been footnotes.

The reader of this book who ended his mathematical studies with the calculus "or even earlier", and who reads conscientiously from cover to cover, might rise a wiser man, but surely a dizzier one. For complete satisfaction, a necessary condition for the reader would be a full understanding of the whole field of mathematics,—but possibly this condition might not be sufficient. There are here many descriptions of various branches of mathematics, most of them necessarily too brief to give the tyro much of an idea as to what it is all about. The graduate student will fare better, but the professional mathematician will probably best appreciate and enjoy this book,—provided the author does not tread on *his* toes!

Those who have read *Men of Mathematics* will know what to expect here in the way of style. After a few drier pages there is always a pungent remark on human frailties, a bit of grim humor, sometimes an aside of a half page or more on what dictators are doing to mathematics, and what philosophers or theologians would do if they could. On the more sober side, it must be granted, in spite of some reservations in the preceding paragraph of this review, that the author has succeeded very well in making his definitions and descriptions of various mathematical systems intelligible to a reader who is not too ill-prepared. "Clear as a Bell" is not always an applicable quotation, however. For example, the reviewer has several times attacked the paragraph that begins at the bottom of page 360 but still cannot determine who said what.

The first chapter, entitled *General Prospectus*, is accessible to all who are likely to read the book. Here the author not only unfolds the plan of the succeeding chapters, but states many of the ideas which he is later to drive home by repeated instances from mathematical history. The following statement is significant:

> "Not all mathematics of the past has survived, even in suitably modernized form. Much has been discarded as trivial, inadequate, or cumbersome, and some has been buried as definitely fallacious. There could be no falser picture of mathematics than that of 'the science which has never had to retrace a step'. If that were true, mathematics would be the one perfect achievement of a race admittedly incapable of perfection. Instead of this absurdity, we shall endeavor to portray mathematics as the constantly growing, human thing that it is, advancing in spite of its errors and partly because of them."

It follows that the validity of a proof is a function of time. In spite of this, mathematics is not a shifting quicksand; it is as stable and as firmly grounded as anything in human experience; often the structure stands, even when its props seem to have gone.

Three great divisions of mathematical history are the Remote Era, to 1637, the Middle Era, to 1800, and the Modern Era. The last is the true golden age, and by far the most prolific; the author asserts that the nineteenth century alone contributed five times as much to mathematics, both as to quantity and power, as had all preceding history. The mathematics of the present century is characterized on page 18.

Mathematics, according to Bell, is always developing, always being revitalized by new conceptions. "Finality is a chimera. Its rare appearances are witnessed by only the mathematically dead".

Not only Chapter I, but also Chapters II-VII, can be read without much technical preparation. Here we have a not too-unorthodox history of mathematics to the year 1687. Chapter II characterizes the pre-Greek period as the Age of Empiricism. Chapter

III, *Firmly Established,* follows Greek mathematics from 600 B. C. to A. D. 300. The Greeks are said first to have recognized that proof by deductive reasoning is the foundation for the structures of number and form, and to have made "the daring conjecture that nature can be understood by human beings through mathematics, and that mathematics is the language most adequate for idealizing the complexity of nature into apprehensible simplicity". Chapters IV and V cover the period to 1200, Chapter VI to the beginning of the seventeenth century, and Chapter VII to 1687.

Beginning with Chapter VIII a new plan is followed. Hereafter each chapter is devoted to the development of a mathematical field or concept, or to an account of impulses received by mathematics from other sciences. In each case the period covered is either the whole or part of the Modern Era. Sometimes the terminus is 1900, sometimes 1940. It is inevitable that the reader should have a feeling of duplication when he thus runs over the same period time after time, and finds the same mathematical system used to illustrate the progress of different concepts. This scheme has, however, unique advantages in bringing into focus the development of ideas, rather than mere chronology.

The number concept is basic in Chapters VIII-XIV. The first of these, somewhat easier to read than the others, gives a general account of extensions of the system of rational integers. The next chapter is on mathematical structure (first defined after twenty-five pages of historical character). We may note here a reference to the tensor calculus; there are others later in the book, but no definition of a tensor. Obviously it would be impossible to define everything within the compass of 600 pages. The wonder is that so much is defined and described, and so successfully. One whose equipment does not go beyond elementary calculus will, however, have many difficult moments in the last three hundred and fifty pages. Not to understand everything that is said in a mathematical argument would, perhaps, irk only the true mathematician. Those who are content to see their mathematics in a golden haze may not be worried.

In each of these chapters we are led from particular, often isolated beginnings along "the usual path to abstractness, generality, and increasing power." There is sharp criticism of the uninspired turner of the crank, in group theory for example. Again and again the author points out how the enormous industry of mathematicians piles up vast accumulations that threaten to choke progress,—and then a way is found around them, or over them, or through them. In various places the mathematics of the twentieth century is characterized. and sometimes cautiously, sometimes fearlessly evaluated. In this connection we observe that the title of Chapter XIII (*From Intuition to Absolute Rigor, 1700-1900*) is ironical.

Chapter XV is devoted to geometry, but one need not expect to find geometry confined to this chapter. For example, topology receives its chief notice much later, in the chapter on invariance (pages 424-436). In fact, it is hard to predict where the history of a given part of mathematics will be found in this book (another instance is the history of elliptic functions in Chapter XVIII, *From Applications to Abstractions*).

Chapters XVI-XVIII concern impulses from science, particularly mechanics, and trace the progress on into abstractness. Then follow chapters on *Differential and Difference Equations, Invariance, Certain Major Theories of Functions, Through Physics to General Analysis and Abstractness: Uncertainties and Probabilities.* In the first part of the last chapter modern progress in mathematical logic is outlined.

The reviewer has found a few errors, of no particular significance. He has agreed with the author "almost everywhere", which does not mean "with no exceptions".

To tell what mathematics is about, how it developed, and what it is doing in the year 1940, so that the comparatively uninformed will be entertained as well as profited, is a difficult undertaking. One lays down the book with admiration for the head that

could assemble all this knowledge and make so much of it useful to the uninitiated, the neophytes, and the adepts.

Northwestern University. D. R. CURTISS

Introductory College Mathematics. Revised edition. By William E. Milne and David R. Davis. Ginn and Co., Boston, 1941. xvi+438+82 pages. $3.00.

The revised edition of this book retains the clearness of presentation and correlation of material that were to be found in the original volume. New chapters have been added, one on Series and two on Analytic Geometry of Space.

The ideas of variables, graphs, rates, tangents, differentiation, and integration are introduced in the first few lessons and used subsequently throughout the book. The amount of material covered is quite large, not all of which could be covered by any class in one year. This is done purposely by the authors in order to provide the flexibility necessary for the various types of courses desired. Several selections for possible courses are suggested in the preface and many others are possible in as much as the last fourteen chapters may be grouped in sections which may be taken in almost any order or omitted at the discretion of the teacher. Some of these chapters the average college freshman would find quite difficult; however, normally they would be taken only by classes of students having exceptional ability.

The individual topics are explained with extreme clarity, and numerous applications are made. However the reviewer would adversely criticize the method of finding the derivatives of $\sin\Theta$ and $\cos\Theta$. Here an interesting but indirect method involving mechanics is used which avoids using

$$\underset{\Theta\to 0}{\text{limit}} \frac{\sin\Theta}{\Theta}.$$

This method is not very convincing and also violates the spirit of the book, in which most problems are solved by direct methods.

The physical make up of the book is excellent, the type clear and figures well done. The tables provided are good and easily read. A large number of exercises, graded as to difficulty, are provided frequently throughout the book. Answers are given to the odd numbered exercises.

This freshman text merits the careful consideration of all college teachers and especially of those who teach the better students.

The University of North Carolina. V. A. HOYLE.

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